Abitur 2015 – B1 – GEOMETRIE

(Aufgabe 1)

a) $E: x_1 + x_3 = 2$

$A(0 | \sqrt{2} | 2)$

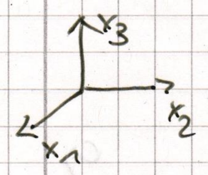
$g: \vec{x} = \vec{a} + \lambda \begin{pmatrix} -1 \\ \sqrt{2} \\ 1 \end{pmatrix}$

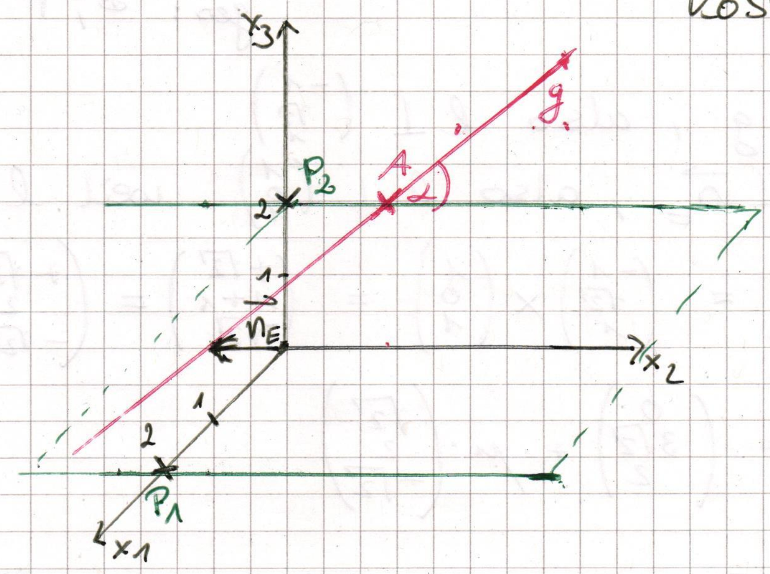
Lage v. E

$g \subset E$

$E \cap x_1$-A, $E \cap x_3$-A

KOSY





$x_1 = 0 \Rightarrow x_3 = 2$

$x_3 = 0 \Rightarrow x_1 = 0$

Die Ebene E verläuft parallel zur x_2-Achse und geht durch die Punkte $P_2(0|0|2)$ und $P_1(2|0|0)$

$g \cap E: \quad x_1 = 0 - \lambda$

$x_2 = \sqrt{2} + \sqrt{2}\lambda$

$x_3 = 2 + \lambda$

in E : $0 - \lambda + 2 + \lambda = 2$

$2 = 2$

allgemein gültig für alle λ

$\Rightarrow g \subset E$

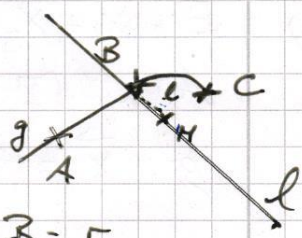
b) x_1x_2-Ebene : $\vec{n} = \begin{pmatrix}0\\0\\1\end{pmatrix}$

$$\cos\beta = \frac{\begin{pmatrix}-1\\\sqrt{2}\\1\end{pmatrix}\circ\begin{pmatrix}0\\0\\1\end{pmatrix}}{\sqrt{1+2+1}\cdot 1} = \frac{1}{2} \Rightarrow \beta = 60^\circ$$

$$\alpha = 90^\circ - 60^\circ = \underline{\underline{30^\circ}}$$

c) $M(0 \mid 3\sqrt{2} \mid 2)$

Lot von M auf g



ges: B; r

Da $l \perp g$, also $l \perp \begin{pmatrix}-1\\\sqrt{2}\\1\end{pmatrix}$

und $l \perp \vec{n}_E$, also $l \perp \begin{pmatrix}1\\0\\1\end{pmatrix}$ weil $l \in E$

$$\Rightarrow \vec{u}_l = \begin{pmatrix}-1\\\sqrt{2}\\1\end{pmatrix} \times \begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}+\sqrt{2}\\1+1\\-\sqrt{2}\end{pmatrix} = \begin{pmatrix}+\sqrt{2}\\2\\-\sqrt{2}\end{pmatrix}$$

$$l: \vec{x} = \begin{pmatrix}0\\3\sqrt{2}\\2\end{pmatrix} + \mu\cdot\begin{pmatrix}\sqrt{2}\\2\\-\sqrt{2}\end{pmatrix}$$

$l \cap g$:

I) $0 - \lambda = \sqrt{2}\mu \Rightarrow \lambda = -\sqrt{2}\mu$

II) $\sqrt{2} + \sqrt{2}\lambda = 3\sqrt{2} + 2\mu$

III) $2 + \lambda = 2 - \sqrt{2}\mu \Rightarrow \lambda = -\sqrt{2}\mu$

$\mathrm{I} = \mathrm{III}$

aus I) $\lambda = -\sqrt{2}\mu$

in II) $\sqrt{2} + \sqrt{2}\cdot(-\sqrt{2}\mu) = 3\sqrt{2} + 2\mu \quad | -\sqrt{2}$

$-2\mu = 2\sqrt{2} + 2\mu$

$-2\sqrt{2} = 4\mu \quad | :4$

$\mu = -\frac{1}{2}\sqrt{2}$

$\lambda = +\sqrt{2}\cdot\frac{1}{2}\sqrt{2} = 1$

$$\Rightarrow \vec{b} = \begin{pmatrix}0\\\sqrt{2}\\2\end{pmatrix} + \begin{pmatrix}-1\\\sqrt{2}\\1\end{pmatrix} = \begin{pmatrix}-1\\2\sqrt{2}\\3\end{pmatrix}, \quad \underline{\underline{B(-1 \mid 2\sqrt{2} \mid 3)}}$$

$$r = |\overrightarrow{MB}| = \left|\begin{pmatrix}-1-0\\2\sqrt{2}-3\sqrt{2}\\3-2\end{pmatrix}\right| = \left|\begin{pmatrix}-1\\-\sqrt{2}\\1\end{pmatrix}\right| = \sqrt{1+2+1} = \underline{\underline{2}}$$

Abi 2015 – B1 – GEO

FS – Aufgabe 1

d) $\vec{v} = \begin{pmatrix} -1 \\ \sqrt{2} \\ 1 \end{pmatrix}$; $|\vec{v}| = \sqrt{1+2+1} = 2 = r$

nach einem Viertelkreis ist man sozusagen um einen Radius in Fahrtrichtung vorwärts gekommen und befindet sich bei einer Rechtskurve um einen Radius weiter rechts, also gegenüber dem Mittelpunkt M (siehe Skizze)

e) $v = 15\frac{m}{s}$ $\qquad$ $1LE = 10m$

$|\vec{AB}| = \left|\begin{pmatrix} -1 - 0 \\ 2\sqrt{2} - \sqrt{2} \\ 3 - 2 \end{pmatrix}\right| = \left|\begin{pmatrix} -1 \\ \sqrt{2} \\ 1 \end{pmatrix}\right| = \sqrt{1+2+1} = 2$

$b = \frac{1}{4} \cdot 2r \cdot \pi = \frac{1}{2} \cdot 2 \cdot \pi = \pi$

Strecke $s = 2 + \pi = 2 + 3{,}14 = 5{,}14$

$s = 51{,}4\ m$

$v = \frac{s}{t} \Rightarrow t = \frac{s}{v}$

$t = \frac{51{,}4m}{15\frac{m}{s}} = \underline{\underline{3{,}4s}}$